On Nash equilibrium feedback solutions to non-cooperative differential games

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Abstract

For non-cooperative differential game, we consider the system of Hamilton-Jacobi equations describing the value functions for the various players, in a Nash equilibrium solution in feedback form. In some cases, when the state space is one-dimensional, this system of PDEs can be analyzed using the theory of hyperbolic conservation laws.

In many cases, however, this system of Hamilton-Jacobi equations is not hyperbolic, hence the Cauchy problem ill posed.

In alternative, one can study the infinite-horizon problem with exponentially discounted cost. The second part of the talk will outline a homotopy approach. The original non-cooperative game is embedded in a family of differential games depending on a parameter $\theta \in [0, 1]$. When $\theta = 0$, the system can be decoupled, and reduced to an optimal control problem for one of the players, while the others adopt a mypic strategy. For $\theta > 0$ we have a genuine non-cooperative game. Conditions are sought which provide the existence of solutions also for $\theta > 0$, or the presence of bifurcations.

An interpretation of this parameterized model will be discussed, in connection with a pricing game between a large producer and a group of small consumers.