

Hölder regularity results for solutions of Hamilton-Jacobi equations

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Abstract

It is well-known that solutions to the basic problem in the calculus of variations, under hypotheses of Tonelli type, may fail to be Lipschitz continuous when the Lagrangian depends on t . Such a lack of Lipschitz regularity is shared by the solutions of the corresponding PDE of dynamic programming.

This phenomenon raises the question of whether such solutions satisfy uniform estimates in some weaker norm. We will show that is indeed the case for a suitable Hölder norm, obtaining uniform estimates for solutions of Hamilton-Jacobi equations and, consequently, for minimizers in the calculus of variations. Our method is based on comparison arguments and a weak reverse Hölder inequality.

Key Words: Hamilton-Jacobi equations, viscosity solutions, Hölder continuity