The Pontryagin maximum principle: half a century on

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Abstract

The talk is aimed to trace the central line in the development of the Pontryagin maximum principle (MP) for optimal control problems with ODEs that have been made during the last half a century. The main topics of the talk are as follows. The control system $\dot{x} = f(t, x, u)$ and the inclusion constraint $u \in U$. The class of admissible control functions L_∞. The canonical problem of Pontryagin type. The needle-type variations of McShane. The Pontryagin maximum principle. An abstract extremum problem with constraints. Dubovitskii–Milyutin scheme and the role of convex structures. Dubovitskii–Milyutin theorem on nonintersection of cones and the stationarity condition. Lagrange multipliers rule in the general smooth and nonsmooth problems. The problems of classical calculus of variations and the Euler–Lagrange equation. Classes of variations in optimal control problems: the uniformly small and needle-type variations, sliding modes, change of time. Proof of the MP: a family of associated problems with smooth constraints parametrized by an index. A compact set of Lagrange multipliers that provide a partial MP for each index. A partial ordering of the indices (comprising a net) and a centered (Alexandrov type) system of compacta. The global MP.

The weak, strong, and Pontryagin minima. State constraints and the corresponding Lagrange multipliers — measures. Adjoint equation with a measure in the right hand side. Conditions for the absence of jump and singular components of the measure. Junction of regimes of different types, the chattering phenomenon (the Fuller and Milyutin examples). Multidimensional chattering. Reformulations of optimal control problems and stability of the Pontryagin MP and Pontryagin extermals. Problems on minimax.

Mixed constraints and their Lagrange multipliers. Iosida–Hewitt theorem. Nonsmoothness of the state and mixed constraints, their directional derivatives and subdifferentials. Regular mixed constraints: positive-linear independence of their gradients w.r.t. control, and a theorem on the absence of singular components. A relaxation theorem for a nonlinear control system with sliding modes. MP for problems with regular mixed constraints. The general optimal control problem with nonregular mixed constraints. Closure w.r.t. measure of a measurable set and a function. The biting lemma for a sequence of integrable functions. The three-storey theorem on almost exact Euler–Lagrange equation. MP for problems with nonregular mixed constraints: the absence of common Lagrange multipliers. Other directions in development and generalization of the Pontryagin MP.