Recent advances in the numerical approximation of pursuit-evasion games via Isaacs equation

EMILIANO CRISTIANI*, MAURIZIO FALCONE†

*UMA, ENSTA
Paris, France
e-mail: emiliano.cristiani@gmail.com

†Dipartimento di Matematica, SAPIENZA - Università di Roma
Rome, Italy
e-mail: falcone@mat.uniroma1.it

Abstract

In this talk we present and analyze a numerical approximation scheme for 2-players Pursuit-Evasion games with state constraints in dimension N. The scheme is based on the dynamic programming approach and the convergence results are obtained in the framework of viscosity solutions.

Our main result shows that the solution of the fully-discrete problem converges to the time-discrete value function as the mesh size Δx goes to zero provided a technical "consistency" assumption on the triangulation is satisfied. Moreover, an *a priori* error bound on that approximation is proved and a very easy sufficient condition guarateeing consistency is shown. Let us remark that, beside the mathematical difficulties, the numerical solution of the Isaacs equation is a very hard task from the computational point of view. In fact, whenever the position of the players is relevant (as in the state constrained problem) and the number of variables can not be reduced we have to solve a first order nonlinear PDE in dimension $2N$. This corresponds to a huge problem in terms of memory allocations and CPU time. In order to overcome this problem we have developed a Fast Marching Methods (FMM). The main feature of these methods is that they do not need to compute the solution everywhere in the domain at every iteration but "follow the flow of the information" starting from the target. At every iteration the FMM algorithm computes the solution *only* in a small neighborhood of the region where the time of capture has been already computed at the previous iterations. The computation proceeds by slices until all the values on the grid have been computed. We will present some preliminary results in this direction.

Key Words: Differential games, Hamilton-Jacobi-Isaacs equations, numerical methods, fast marching methods