

# The Pontryagin derivative in optimal control

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## Abstract

A basic feature of the maximum principle is its native Hamiltonian format, inherent in the principle regardless of any regularity conditions imposed on the optimal problem under consideration. It assigns canonically to the problem a family of Hamiltonian systems, indexed with the control parameter, and complements the system with the maximum condition, which makes it possible to solve the initial value problem for the system, "dynamically" eliminating the parameter as we proceed along the trajectory, thus providing the extremals of the problem.

Much was said about the maximum condition since its discovery in 1956, and all achievements in the field were mainly credited to it, whereas the Hamiltonian format of the maximum principle was always taken for granted and never discussed seriously.

Meanwhile the very possibility of formulating the maximum principle is intimately connected with its native Hamiltonian format and with the parametrization of the problem with the control parameter.

Both these starting steps were made by L. S. Pontryagin in 1956 on a completely empty ground, in fact, out of nothing, and eventually led to the discovery of the maximum principle.

Taking into account the nature of our Conference dedicated to the 50-th Anniversary of Pontryagin's maximum principle and considering that the current 2008 year is the centennial birthday of Lev Semenovich Pontryagin, I decided to return to this, now semi-historical, topic and to give a short exposition of the Hamiltonian format of the maximum principle.