# Nonsmooth Analysis and Discontinuous Feedback: Applications to Optimal Team Pursuit

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**Abstract**

The problem of coordinated control of a team of mobile objects presents one of the challenges of the modern control theory. In this talk we demonstrate mathematical techniques based on nonsmooth analysis and theory of discontinuous feedback which are applied to find an optimal solution of one of such coordinated control problems – the problem of team pursuit.

Consider a problem of pursuit of one evader $x_0$ by a team of pursuers $x_i, i = 1, \dots, m$ in $n$-dimensional space $R^n$. It is assumed that the dynamics of the players is *simple*, namely, it is described by the following equations

$$\dot{x}_0 = u_0, \quad \dot{x}_1 = u_1, \quad \dots, \quad \dot{x}_m = u_m$$

and controls $v, u_1, \dots, u_m$ satisfy constraints $\|u_0\| \le \sigma_0$, $\|u_1\| \le \sigma_1, \dots, \|u_m\| \le \sigma_m$. The pursuit is over at some moment $T$ if there exists an index $i$ such that $\|x_0(T) - x_i(T)\| \le \ell_i$ where $\ell_i$ are some nonnegative constants. We are interested in finding the optimal pursuit time.

To find such optimal pursuit time we consider some auxiliary mathematical programming problem and its value function. Modern tools of nonsmooth analysis (in particular, proximal calculus) are used to demonstrate that this value function is also a nonsmooth value function of the original differential game of team pursuit.

It is shown that this nonsmooth value function can be used to design optimal feedback controls for pursuers $k_1(x), \dots, k_m(x)$. It can also be used to study robustness properties of these discontinuous optimal feedback controls with respect to small measurement and actuators errors and external disturbances.

**Key Words:** Optimal Feedback, Differential Games, Group Pursuit, Nonsmooth Analysis, Robust Feedback