Theory, Numerics and Applications of Optimal Control Problems with Control and State Delays

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Abstract

Optimal control problems with delays in control and state variables arose nearly 40 years ago in chemical process control. Since then, theory and numerics have made only moderate progress towards a systematic study of the different control classes, e.g., controls satisfying the strict Legendre-condition or bang-bang and singular controls. In particular, mixed control-state or pure state inequality constraints have not been considered in full generality. In this talk, we derive a Pontryagin type Minimum Principle for retarded control problems with control and state constraints. Under the assumption that the ratio of the control and state delay is a rational number, a simple proof can be devised by augmenting the retarded control problem to a standard optimal control problem. This technical assumption is crucial for the design of numerical methods which are based upon discretisation and nonlinear programming techniques. Two challenging examples illustrate theory and numerics: (1) a continuous stirred tank reactor problem in chemical engineering; (2) an application in biomedicine: computation of optimal multi-drug protocols in a generic model of the innate immune response.