On a class of optimal control problems arising within mathematical programs with equilibrium constraints

JIŘÍ V. OUTRATA*

*Institute of Information Theory and Automation,
Academy of Sciences of the Czech Republic,
18208 Prague, Czech Republic
e-mail: outrata@utia.cas.cz

Abstract

The lecture deals with *mathematical programs with equilibrium constraints* (MPECs) of the form

$$\begin{array}{ll} \text{minimize} & f(x,y) \\ \text{subject to} & \\ & 0 \in F(x,y) + Q(y) \\ & x \in \omega \\ & y \in \Xi, \end{array} \tag{1}$$

where x is the *control* variable, y is the *state* variable and f is the *objective*.

The most important part of this model is the *generalized equation*

$$0 \in F(x,y) + Q(y), \tag{2}$$

where F is a continuously differentiable map and Q is a closed-graph multifunction. We will assume that the solution map S, defined by

$$S(x) := \{y \,|\, 0 \in F(x,y) + Q(y)\},$$

is single-valued and locally Lipschitz. Therefore (1) amounts to a special optimal control problem which will be investigated both in the finite-dimensional as well as in an infinite-dimensional setting. Our main workhorse is the so-called implicit programming approach based on the reformulation of (1) to the optimization problem

$$\begin{array}{ll} \text{minimize} & f(x,S(x)) + \delta_{\Xi}(S(x)) \\ \text{subject to} & \\ & x \in \omega \end{array} \tag{3}$$

in the control variable only.

The first part of the talk is devoted to first-order necessary optimality conditions in the finite-dimensional case.

The second part of the talk deals with the possibility of the numerical solution of (3) via the penalized problem

$$\begin{array}{ll} \text{minimize} & f(x, S(x)) + R\,\mathrm{dist}_{\Xi}(S(x)) \\ \text{subject to} & \\ & x \in \omega, \end{array} \tag{4}$$

with a positive penalty parameter R. The question of the computation of subgradients of the composite objective in (4) is crucial in this connection.

The last part of the talk concerns the optimality conditions for (1) in the case of equilibria governed by a linear elliptic variational inequality over the Sobolev space $\overset{\circ}{H}{}^1(\Omega)$.

Key Words: equilibrium constraint, optimality conditions, calmness.