State Constrained Optimal Control of Coupled Systems of ODEs and PDEs with Applications to Hypersonic Flight

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Abstract

During ascent and reentry of a hypersonic space vehicle into the atmosphere of any heavenly body, the space vehicle is subjected to extreme aerothermic loads. Therefore an efficient, sophisticated and lightweight thermal protection system is determinative for the success of the entire mission. For a deeper understanding of the conductive, convective and radiative heating effects through a thermal protection system, a mathematical model is investigated which is given by an optimal control problem subject to not only the usual dynamic equations of motion and suitable control and state variable inequality constraints for the ODE part of the problem, but also subject to an instationary quasi-linear heat equation with nonlinear boundary conditions and a state constraint, forming the PDE part of the problem. By this model the temperature of the heat shield can be limited in certain critical regions.

The resulting ODE-PDE constrained optimal control problem is, because of its complexity, solved by the approach *First discretize, then optimize*. The discretization scheme used is a second-order finite-volume scheme in space for the semi-discretization of the quasi-linear parabolic partial differential equation in its space variables. This yields a large scale nonlinear ODE constrained optimal control problem with multiple state and/or control variable inequality constraints, in particular for the limitation of the heat load. This problem is also solved by the approach *First discretize, then optimize* and ends in a large scale non-linear programming problem.

Numerical results are presented. They show, that the aerothermic load and the fuel loss can be considerably reduced by optimization.

Some theoretical light on this problem is given by the investigation of an academic problem of an equivalent structure, which is based on the well-known rocket car problem, a simplification of the classical harmonic oscillator problem, which has illustrated the development of the maximum principle. This problem is enriched with a linear heat equation and therefore is called, tongue-in-cheek, the hypersonic rocket car problem. This at the first glance simple staggered state-constrained ODE-PDE optimal problem is surprisingly complicated and neither analytically solvable nor numerically solvable by standard methods, although each part has an analytical solution. In particular, this is caused by the fact that the state constraint is not local in ODE terms, or, alternatively, the PDE formulation of the problem exhibits both distributed control terms of non-standard form and isoperimetrical side conditions. Nevertheless the discrete adjoints of a *First discretize, then optimize* approach allows the approximate verification of the most important first order necessary conditions in particular the maximum principle.

Key Words: Optimal control of systems of ODEs and PDEs, non-local state constraints, first discretize — then optimize, approximate verification of first order necessary conditions