Integrability of optimal controls in sub-Riemannian problems

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Abstract

It is well known that studying the geodesic equation for sub-Rimannian spaces leads to an optimal control problem for systems that are linear with respect to controls and with a quadratic cost. We study the integrability properties of the hamiltonian system defined for such system by the Pontryagin Maximum Principle.

For homogeneous sub-Riemannian spaces the adjoint equation (defining the optimal controls) can be separated from that describing the optimal curves. In the 3-dimensional case, we show deep relations between the geometry of the sub-Riemannian spaces and integrability properties of its adjoint equations. In particular, the space is symmetric if and only if the optimal controls are elliptic functions of time which, in turn, is equivalent to the existence of two quadratic first integrals. We also discuss nonintegrability of the adjoint equation for homogeneous but non symmetric sub-Riemannian spaces. Then we show, using the Morales-Ramis theory, nonintegrability (in the category of meromorphic functions) of the systems defining optimal controls (geodesic equations) in 3-dimensional non homogenous nilpotent case and in the n-dimensional case on the rotation group $SO(n)$.

Key Words: integrability, optimal controls, sub-Riemannian geometry, geodesic equation