Second order conditions and Hamiltonian flows

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Abstract

The aim of the talk is to explain the role of Hamiltonian flows in the study of second order conditions for optimal control problems.

The crucial point is the possibility of defining an *intrinsic second variation*, i.e. a linear–quadratic (LQ) problem on a vector space which is coordinate–free and hence can be defined even if the problem is stated on a manifold. It turns out that the Hamiltonian associated to the second variation is strictly linked to the maximized Hamiltonian associated to the original problem and this makes it possible to give a geometric interpretation of the signature of the second variation and to prove sufficient conditions.

In particular, second order conditions for reference trajectories including singular and bang arcs lead to the study of LQ forms on Hilbert spaces which include real parameters entering in the definition in a non–standard way. To be more precise we are lead to consider a quadratic form I defined on a subspace V of the Hilbert space $R^k \times L^2([0,T], R^m)$ of the following kind.

- V is the space of the couples $(\alpha, u) \in R^k \times L^2([0,T], R^m)$ satisfying

$$\dot{\eta}(t) = B(t)u(t)\ , \quad a.e.\ t \in [0,T]$$
$$\begin{pmatrix} \eta(0) \\ \eta(T) \end{pmatrix} = M\alpha$$

- I is defined by

$$I(\alpha, u) = \frac{1}{2}\alpha^\top \Gamma_0 \alpha + \Big(\eta^\top(0), \eta^\top(T)\Big)\Gamma_1 \begin{pmatrix} \eta(0) \\ \eta(T) \end{pmatrix} + \\ + \frac{1}{2}\int_0^T \Big\{ 2u^\top(s)Q(s)\eta(s) + u^\top(s)R(s)u(s) \Big\}\, ds.$$

The talk will address some cases exploiting the Hamiltonian approach to the study of the signature (and hence to the necessary conditions) and to the proof of sufficient conditions.

Key Words: second order conditions, Hamiltonian approach, singular arcs, bang arcs