Taming functions with unbounded variations in optimal control.

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Abstract

We discuss three areas of optimal control in which functions with unbounded variations appear in important problems. The first one is in well known problems of classical optimal control in which one approaches optimal solutions with rapidly oscillating control functions. A limit solution is then obtained by imbedding the controls in the class of relaxed controls which are functions with values that are probability measures. The second area is one dealing with conflicting control games (defined by differential equations) in which the defending party must devise a defending control which will ensure the best results possible when faced with any attack strategy; this is thus a min-max problem subject to restrictions. In the simpler of such problems like evasion problems, replacing each defense and attack control with corresponding relaxed controls will suffice. In more complicated problems (in which the opposing controls are non-additively coupled) one must resort to functions whose values are joint probability measures for both sides that give rise to ordinary relaxed controls for the defense and hyperrelaxed controls (related to conditional probabilities) for the offense. In the third area under discussion, one dealing with modeling of discontinuities in state functions, most researchers used finite measures to describe the problem but in so doing lost much information contained in the physical approximations. Our approach was to reparametrize the problem by dealing with the graphs of the state functions, and replacing time, the original independent variable, with a new independent variable akin to the length along the graph.