# Chattering control in infinite-dimensional spaces

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**Abstract**

Consider the control system

$$z_{tt} + Bz = C(x)u.$$

Here

$$t \in R,\ x \in \Omega \subset R^n,\ z : R \times \Omega \to R^k. \tag{1}$$

The control function $u$ is a scalar, $|u| \leq 1$. The operator $B$ is an elliptic differential operator in the domain $\Omega$. As a boundary value problem we consider

$$z(0, x) = z_0(x),\ z_t(0, x) = z_1(x);\ G|_{\partial\Omega} = 0$$

where $G$ is a boundary operator that provides self-conjugacy of $B$; the spectrum of $B$ is $\{\lambda_i\}$, eigenfunctions generate a basis in $l_2$. We seek the control function $u(x)$ that minimize the functional

$$\int_0^\infty \int_\Omega |z(t, x)|^2 dxdt.$$

Expanding by eigenfunctions of $B$ one obtains the problem of simultaneous stabilization of oscillators by the same scalar control that acts on each oscillator. So one has the control problem in the space $l_2$

$$\int_0^\infty \sum_{i=1}^\infty s_i^2(t)dt \to inf, \quad \frac{d^2 s_i}{dt^2} + \lambda_i s_i = c_i u, \quad (i = 1, 2...), \quad |u| \leq 1.$$

We proved:

- the necessary conditions for optimality (Pontryagin's maximum principle)
- the sufficiency theorem (the functional in question is convex)
- the existence and the uniqueness theorem

It was found the manifold $S$ consisting of singular trajectories. It was proved the theorem on bundles that claims: There exists a bundle over the base $S$ with two-dimensional fibres filled by chattering trajectories, that is by trajectories reaching $S$ in finite time interval with infinite number of switches. So we design chattering synthesis in infinite-dimensional space.

As an example we investigate the problem of mean square deviation of the elastic Timoshenko beam from the desirable position.We consider the beam that is clamped at one end to a rotating disc and with free other end. We supposed that the beam is controlled by a bounded in absolute value torque acting on the disc. Using the theorem on bundles we found the optimal synthesis.